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Radio Geometric Mean Number of Splitting Of Star and Bistar

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Abstract:
A radio Geometric Mean Labeling of a connected graph G is a one to one map f from the vertex set V(G) to the set of natural numbers N such that for two distinct vertices u and v of G, $d(u,v)+\left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 1+diam(G)$. The radio geometric mean number of $f, r_{gmn}(f)$ is the maximum number assigned to any vertex of G. The radio geometric mean number of G, $r_{gmn}(G)$ is the minimum value of $r_{gmn}(f)$ taken over all radio geometric mean labeling f of G. In this paper, we determine the radio geometric mean number of splitting graph of star and bistar.

***Keywords**: Radio Geometric Mean labeling, Star, Bistar, Diameter.*

1. INTRODUCTION

We consider finite, simple, undirected graphs only. Let V(G) and E(G) respectively denote vertex set and edge set of G. Chartand et al.[1] defined the concept radio labeling of G in 2001. Radio labeling of graphs is applied in channel assignment problem [1]. Radio number of several graphs determined [2,7,5,9]. In this sequence Ponraj et al.[8] introduced the radio mean labeling in G. Here we introduce a new type of labeling, a radio geometric mean labeling is a one to one mapping f from V(G) to N satisfying the condition

$$d(u,v)+\left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 1+diam(G)$$

for every $u,v \in V(G)$.

The span of a labeling f is the maximum integer that f maps to a vertex of graph G. The radio geometric mean number of G, r_{gmn}(G) is the lowest span taken over all radio geometric mean labeling of the graph G. In this paper we determine the radio geometric mean number of some star like graphs. Let *x* be any real number. Then $\lceil x \rceil$ stands for smallest integer greater than or equal to *x*. Terms and definitions not defined here are followed from Harary [12] and Gallian [13].

The channel assignment to radio transmitters is one of the main objectives in setup of wireless communication system. A proper channel assignment to radio transmitters which satisfies interference constraints with maximum use of spectrum is a need of wireless communication system. The interference constraints between a pair of transmitters is closely related with separation of channels and distance between transmitters. In a network, if two transmitters are closer then higher the interference between them and large separation.

Definition 1.1 A **Star** is the complete bipartite graph $K_{1,n}$.

Definition 1.2 The graph **Bistar** $B_{n,n}$ obtained by joining the center vertices of two copies of $K_{1,n}$ with an edge.

Definition 1.3 [14] For a graph G, the split graph is obtained by adding to each vertex *v* a new vertex *v'* such that *v'* is adjacent to every vertex that is adjacent to *v* in G. The resultant graph is denoted as *Spl(G)*.

2. MAIN RESULTS

Theorem 2.1 Radio Geometric Mean number of Splitting of star, $r_{gmn}\left(Spl\left(K_{1,n}\right)\right)=2\left(n+1\right)$.

Proof: Let G be a $Spl\left(K_{1,n}\right)$ with *2(n+1)* vertices and *3n* edges.

The diameter of $Spl\left(K_{1,n}\right), n>1=3$.

Let $v_1, v_2, v_3, ..., v_n$ be the pendant vertices and *v* be the apex vertex of $K_{1,n}, n>1$ and

$u, u_1, u_2, u_3, ..., u_n$ be added vertices corresponding to $v, v_1, v_2, v_3, ..., v_n$ to obtain $Spl(K_{1,n})$.
We define the labeling *f* as follows,

$$f(u) = 2(n+1)$$
$$f(v_i) = i \quad ;1 \le i \le n$$
$$f(v) = 2n+1$$
$$f(u_i) = n+i \quad ;1 \le i \le n$$

Now we check the radio geometric mean condition for any two vertices, it should satisfy

$$d(u,v) + \left\lceil \sqrt{f(u)f(v)} \right\rceil \ge 1 + diam(G) = 1+3 = 4$$

Case (i): Check the pair (u, v_i)

$$d(u,v_i) + \left\lceil \sqrt{f(u)f(v_i)} \right\rceil \ge 1 + \left\lceil \sqrt{2(n+1).(1)} \right\rceil \ge 4$$

Case (ii): Check the pair (u, v)

$$d(u,v) + \left\lceil \sqrt{f(u)f(v)} \right\rceil \ge 2 + \left\lceil \sqrt{2(n+1).(2n+1)} \right\rceil \ge 8$$

Case (iii): Check the pair (u, u_i)

$$d(u,u_i) + \left\lceil \sqrt{f(u)f(u_i)} \right\rceil \ge 3 + \left\lceil \sqrt{2(n+1).(n+1)} \right\rceil \ge 8$$

Case (iv): Verify the pair (u_i, v_j)

Subcase (i): If $i = j$

$$d(u_i,v_j) + \left\lceil \sqrt{f(u_i)f(v_j)} \right\rceil \ge 2 + \left\lceil \sqrt{(1)(n+1)} \right\rceil \ge 4$$

Subcase (ii): If $i \ne j$

$$d(u_i,v_j) + \left\lceil \sqrt{f(u_i)f(v_j)} \right\rceil \ge 2 + \left\lceil \sqrt{(2)(n+1)} \right\rceil \ge 5$$

Case (v): Verify the pair (u_i, u_j), $i \ne j$

$$d(u_i,u_j) + \left\lceil \sqrt{f(u_i)f(u_j)} \right\rceil \ge 2 + \left\lceil \sqrt{(n+1)(n+2)} \right\rceil \ge 6$$

Case (vi): Verify the pair (v_i, v_j), $i \ne j$

$$d(v_i,v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \ge 2 + \left\lceil \sqrt{(1)(2)} \right\rceil \ge 4$$

Case (vii): Check the pair (v, v_i)

$$d(v,v_i) + \left\lceil \sqrt{f(v)f(v_i)} \right\rceil \ge 1 + \left\lceil \sqrt{(2n+1).(1)} \right\rceil \ge 4$$

Case (viii): Check the pair (v, u_i)

$$d(v,u_i) + \left\lceil \sqrt{f(v)f(u_i)} \right\rceil \ge 1 + \left\lceil \sqrt{(2n+1).(n+1)} \right\rceil \ge 5$$

Hence $r_{gmn}\left(Spl\left(K_{1,n}\right)\right) = 2(n+1)$, $n > 1$.

Theorem 2.2 Radio Geometric Mean number of Splitting of bistar, $r_{gmn}\left(Spl\left(B_{n,n}\right)\right) = 4(n+1)$.

Proof: Consider $B_{n,n}$ with the vertex set $\{u, v, u_i, v_i : 1 \le i \le n\}$ where u_i, v_i are the pendant vertices. In order to obtain $Spl(B_{n,n})$ add u', v', u_i', v_i' vertices corresponding to u, v, u_i, v_i where $1 \le i \le n$.
$|V(G)| = 4(n+1)$ and $|E(G)| = 3(2n+1)$.
The diameter of the splitting of bistar is 3.
We define the labeling *f* as follows,
Assign the labels of the vertices u, v, u_i, v_i be

$$f(u) = 4(n+1)$$
$$f(v) = 4n+2$$
$$f(u_i) = 2i-1 \quad ; 1 \le i \le n$$
$$f(v_i) = 2i \quad ; 1 \le i \le n$$

and the labels of the vertices u', v', u_i', v_i' be

$$f(u') = 4n+3$$
$$f(v') = 4n+1$$
$$f(u_i') = 2n+2i \quad ; 1 \le i \le n$$
$$f(v_i') = 2n+2i-1 \quad ; 1 \le i \le n$$

Now we check the radio geometric mean condition for any two vertices, it should satisfy

$$d(u,v) + \left\lceil \sqrt{f(u)f(v)} \right\rceil \ge 1 + diam(G) = 1+3 = 4$$

Case (1): Check the pair (u, u')

$$d(u,u') + \left\lceil \sqrt{f(u)f(u')} \right\rceil \ge 2 + \left\lceil \sqrt{4(n+1).(4n+3)} \right\rceil \ge 10$$

Case (2): Check the pair (u, u_i)

$$d(u,u_i) + \left\lceil \sqrt{f(u)f(u_i)} \right\rceil \ge 1 + \left\lceil \sqrt{4(n+1).(1)} \right\rceil \ge 4$$

Case (3): Check the pair (u, u_i')

$$d(u,u_i') + \left\lceil \sqrt{f(u)f(u_i')} \right\rceil \ge 1 + \left\lceil \sqrt{4(n+1).(2n+2)} \right\rceil \ge 7$$

Case (4): Verify the pair (u_i, u_j), $i \ne j$

$d(u_i,u_j)+\left\lceil\sqrt{f(u_i)f(u_j)}\right\rceil \geq 2+\left\lceil\sqrt{(1)(3)}\right\rceil \geq 4$

Case (5): Verify the pair $(u_i',u_j'),\ i\neq j$

$d(u_i',u_j')+\left\lceil\sqrt{f(u_i')f(u_j')}\right\rceil \geq 2+\left\lceil\sqrt{(2n+2)(2n+4)}\right\rceil \geq 7$

Case (6): Verify the pair (u_i,u_j')

Subcase (i): If $i=j$

$d(u_i,u_j')+\left\lceil\sqrt{f(u_i)f(u_j')}\right\rceil \geq 2+\left\lceil\sqrt{(1)(2n+2)}\right\rceil \geq 4$

Subcase (ii): If $i\neq j$

$d(u_i,u_j')+\left\lceil\sqrt{f(u_i)f(u_j')}\right\rceil \geq 2+\left\lceil\sqrt{(2)(2n+2)}\right\rceil \geq 5$

Case (7): Check the pair (u',u_i)

$d(u',u_i)+\left\lceil\sqrt{f(u')f(u_i)}\right\rceil \geq 2+\left\lceil\sqrt{(4n+3).(1)}\right\rceil \geq 5$

Case (8): Check the pair (u',u_i')

$d(u',u_i')+\left\lceil\sqrt{f(u')f(u_i')}\right\rceil \geq 1+\left\lceil\sqrt{(4n+3).(2n+2)}\right\rceil \geq 7$A

Case (9): Check the pair (v,v')

$d(v,v')+\left\lceil\sqrt{f(v)f(v')}\right\rceil \geq 2+\left\lceil\sqrt{(4n+1).(4n+2)}\right\rceil \geq 8$

Case (10): Check the pair (v,v_i)

$d(v,v_i)+\left\lceil\sqrt{f(v)f(v_i)}\right\rceil \geq 1+\left\lceil\sqrt{(4n+2).(2)}\right\rceil \geq 5$

Case (11): Check the pair (v,v_i')

$d(v,v_i')+\left\lceil\sqrt{f(v)f(v_i')}\right\rceil \geq 1+\left\lceil\sqrt{(4n+1).(4n+2)}\right\rceil \geq 7$

Case (12): Verify the pair $(v_i,v_j),\ i\neq j$

$d(v_i,v_j)+\left\lceil\sqrt{f(v_i)f(v_j)}\right\rceil \geq 2+\left\lceil\sqrt{(2)(4)}\right\rceil \geq 5$

Case (13): Verify the pair $(v_i',v_j'),\ i\neq j$

$d(v_i',v_j')+\left\lceil\sqrt{f(v_i')f(v_j')}\right\rceil \geq 2+\left\lceil\sqrt{(2n+1)(2n+3)}\right\rceil \geq 8$

Case (14): Verify the pair (v_i,v_j')

Subcase (i): If $i=j$

$d(v_i,v_j')+\left\lceil\sqrt{f(v_i)f(v_j')}\right\rceil \geq 2+\left\lceil\sqrt{(2)(2n+1)}\right\rceil \geq 5$

Subcase (ii): If $i\neq j$

$d(v_i,v_j')+\left\lceil\sqrt{f(v_i)f(v_j')}\right\rceil \geq 2+\left\lceil\sqrt{(2)(2n+3)}\right\rceil \geq 6$

Case (15): Check the pair (v',v_i)

$d(v',v_i)+\left\lceil\sqrt{f(v')f(v_i)}\right\rceil \geq 2+\left\lceil\sqrt{(4n+1).(2)}\right\rceil \geq 6$

Case (16): Check the pair (v',v_i')

$d(v',v_i')+\left\lceil\sqrt{f(v')f(v_i')}\right\rceil \geq 1+\left\lceil\sqrt{(4n+1).(2n+1)}\right\rceil \geq 5$

Case (17): Check the pair (u,v)

$d(u,v)+\left\lceil\sqrt{f(u)f(v)}\right\rceil \geq 1+\left\lceil\sqrt{4(n+1).(4n+2)}\right\rceil \geq 8$

Case (18): Check the pair (u,v_i')

$d(u,v_i')+\left\lceil\sqrt{f(u)f(v_i')}\right\rceil \geq 1+\left\lceil\sqrt{4(n+1).(2n+1)}\right\rceil \geq 6$

Case (19): Check the pair (u,v_i)

$d(u,v_i)+\left\lceil\sqrt{f(u)f(v_i)}\right\rceil \geq 2+\left\lceil\sqrt{4(n+1).(2)}\right\rceil \geq 6$

Case (20): Verify the pair (u,v')

$d(u,v')+\left\lceil\sqrt{f(u)f(v')}\right\rceil \geq 2+\left\lceil\sqrt{4(n+1)(4n+1)}\right\rceil \geq 9$

Case (21): Verify the pair (u',v)

$d(u_i',u_j')+\left\lceil\sqrt{f(u_i')f(u_j')}\right\rceil \geq 2+\left\lceil\sqrt{(2n+2)(2n+4)}\right\rceil \geq 7$

Case (22): Verify the pair (u',v_i')

$d(u',v_i')+\left\lceil\sqrt{f(u')f(v_i')}\right\rceil \geq 2+\left\lceil\sqrt{4(n+1).(2)}\right\rceil \geq 6$

Case (23): Check the pair (u',v_i)

$d(u',u_i)+\left\lceil\sqrt{f(u')f(u_i)}\right\rceil \geq 2+\left\lceil\sqrt{(4n+3).(2)}\right\rceil \geq 6$

Case (24): Check the pair (u',v')

$$d(u',v')+\left\lceil\sqrt{f(u')f(v')}\right\rceil \geq 3+\left\lceil\sqrt{(4n+3).(4n+1)}\right\rceil \geq 7$$

Case (25): Verify the pair (u_i',v_j')

Subcase (i): If $i=j$

$$d(u_i',v_j')+\left\lceil\sqrt{f(u_i')f(v_j')}\right\rceil \geq 3+\left\lceil\sqrt{(2n+2)(2n+1)}\right\rceil \geq 7$$

Subcase (ii): If $i\neq j$

$$d(u_i',v_j')+\left\lceil\sqrt{f(u_i')f(v_j')}\right\rceil \geq 3+\left\lceil\sqrt{(2n+2)(2n+3)}\right\rceil \geq 10$$

Case (26): Verify the pair (u_i',v)

$$d(u_i',v)+\left\lceil\sqrt{f(u_i')f(v)}\right\rceil \geq 2+\left\lceil\sqrt{(2n+2)(4n+2)}\right\rceil \geq 7$$

Case (27): Verify the pair (u_i',v_j)

Subcase (i): If $i=j$

$$d(u_i',v_j)+\left\lceil\sqrt{f(u_i')f(v_j)}\right\rceil \geq 3+\left\lceil\sqrt{(2n+2)(2)}\right\rceil \geq 5$$

Subcase (ii): If $i\neq j$

$$d(u_i',v_j)+\left\lceil\sqrt{f(u_i')f(v_j)}\right\rceil \geq 3+\left\lceil\sqrt{(2n+2)(4)}\right\rceil \geq 8$$

Case (28): Verify the pair (u_i',v')

$$d(u_i',v')+\left\lceil\sqrt{f(u_i')f(v')}\right\rceil \geq 2+\left\lceil\sqrt{(2n+2)(4n+1)}\right\rceil \geq 7$$

Case (29): Verify the pair (u_i,v_j')

Subcase (i): If $i=j$

$$d(u_i,v_j')+\left\lceil\sqrt{f(u_i)f(v_j')}\right\rceil \geq 3+\left\lceil\sqrt{(1)(2n+1)}\right\rceil \geq 5$$

Subcase (ii): If $i\neq j$

$$d(u_i,v_j')+\left\lceil\sqrt{f(u_i)f(v_j')}\right\rceil \geq 3+\left\lceil\sqrt{(1)(2n+3)}\right\rceil \geq 6$$

Case (30): Verify the pair (u_i,v)

$$d(u_i,v)+\left\lceil\sqrt{f(u_i)f(v)}\right\rceil \geq 2+\left\lceil\sqrt{(1)(4n+2)}\right\rceil \geq 5$$

Case (31): Verify the pair (u_i,v')

$$d(u_i,v')+\left\lceil\sqrt{f(u_i)f(v')}\right\rceil \geq 2+\left\lceil\sqrt{(1)(4n+1)}\right\rceil \geq 5$$

Case (32): Verify the pair (u_i,v_j)

Subcase (i): If $i=j$

$$d(u_i,v_j)+\left\lceil\sqrt{f(u_i)f(v_j)}\right\rceil \geq 3+\left\lceil\sqrt{(1)(2)}\right\rceil \geq 5$$

Subcase (ii): If $i\neq j$

$$d(u_i,v_j)+\left\lceil\sqrt{f(u_i)f(v_j)}\right\rceil \geq 3+\left\lceil\sqrt{(3)(2)}\right\rceil \geq 6$$

Hence every pair of vertices satisfies the radio geometric mean condition.

Thus $r_{gmn}\left(Spl\left(B_{n,n}\right)\right)=4\left(n+1\right)$.

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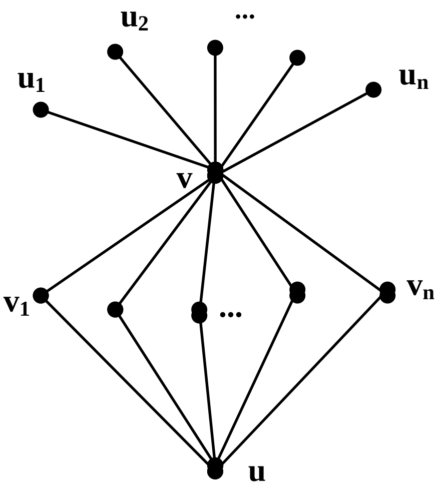
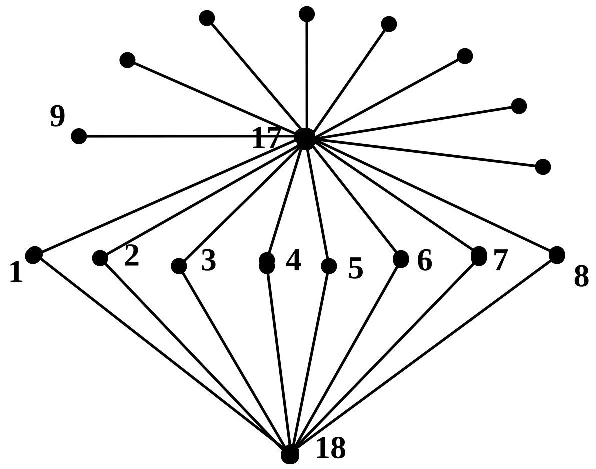
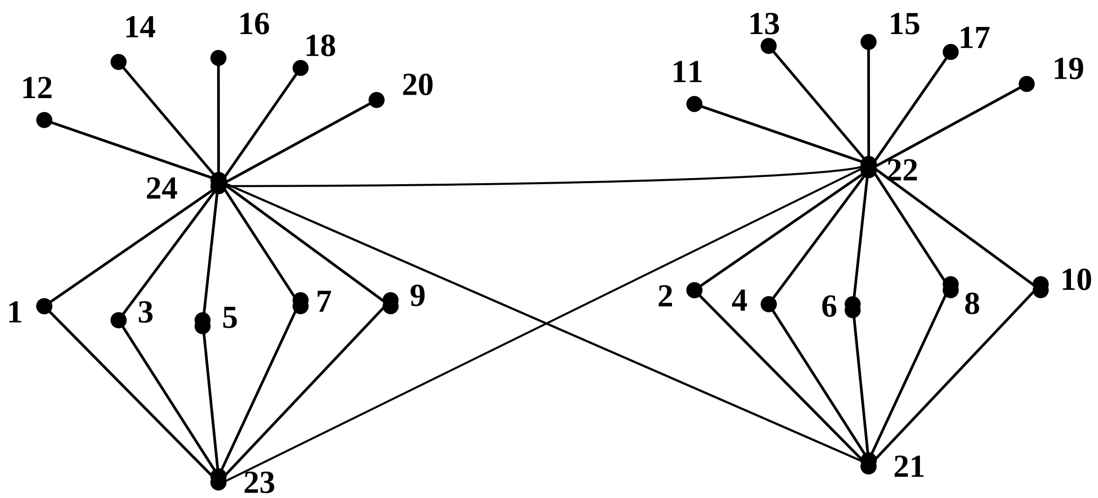
Illustrations:



Spl ($K_{1,n}$) General Graph Labeling



RGML of Spl ($K_{1,8}$)



RGML of Spl ($B_{5,5}$)